Find a homogeneous linear differential equation with constant coefficients whose general solution is $y = Ae^t + Be^t\cos 3t + Ce^t \sin 3t$. Write your final answer using $y', y'', \ldots$ notation, not D notation. **SCORE: _____ / 4 PTS**

$r = 1, \underline{1 \pm 3i}$

$(r-1)((r-1)^2+3^2)=0$

$\underline{(r-1)}\underline{(r^2-2r+10)}=0$

$r^3-3r^2+12r-10=0$

$\underline{y'''-3y''+12y'-10y=0}$

EACH UNDERLINED ITEM WORTH 1 POINT UNLESS OTHERWISE INDICATED

Find the general solution of $4x^2y''+5y=0$ on the interval $(0,\infty)$. **SCORE: _____ / 4 PTS**

$\underline{4r^2-4r+5=0}$

$r = \frac{4\pm\sqrt{-64}}{8}$

$= \frac{4\pm 8i}{8}$

$= \underline{\frac{1}{2} \pm i}$

$y = \underline{Ax^{\frac{1}{2}}\cos \ln x} + \underline{Bx^{\frac{1}{2}}\sin \ln x}$

Find the general solution of $3y'''+8y''+7y'+2y=0$. **SCORE: _____ / 6 PTS**

$3r^3+8r^2+7r+2=0$

$$\begin{array}{r|rrrr} -1 & 3 & 8 & 7 & 2 \\ & & -3 & -5 & -2 \\ \hline & 3 & 5 & 2 & \boxed{0} \end{array}$$

$\underline{(r+1)(3r^2+5r+2)=0}$, $1\frac{1}{2}$

$\underline{(r+1)(3r+2)(r+1)=0}$ $1\frac{1}{2}$

$r=-1,-1,-\frac{2}{3}$

$y = \underline{Ae^{-t}} + \underline{Bte^{-t}} + \underline{Ce^{-\frac{2}{3}t}}$ OK IF YOU USED X OR t

$y_p = 1 - 4x$ is a particular solution of $y'' - 2y' - 8y = 32x$. **SCORE: _____ / 9 PTS**

[a] Using linearity, find a particular solution of $y'' - 2y' - 8y = 16x$.

$16x = \frac{1}{2}(32x) \rightarrow y = \frac{1}{2}(1-4x) = \underline{\frac{1}{2} - 2x}$

[b] By inspection, find a particular solution of $y'' - 2y' - 8y = 1$.

$-8y = 1 \longrightarrow y = \underline{-\frac{1}{8}}$

[c] Using superposition and linearity, find a particular solution of $y'' - 2y' - 8y = 16x - 4$.

$y = \frac{1}{2} - 2x - 4\left(-\frac{1}{8}\right) = \underline{1 - 2x}$

[d] Find the general solution of $y'' - 2y' - 8y = 16x - 4$.

$r^2 - 2r - 8 = 0$

$(r-4)(r+2) = 0$

$r = 4, -2$

$y = \underline{1 - 2x + \overline{Ae^{4x} + Be^{-2x}}}$

FOR SOLUTION TO HOMOGENEOUS EQUATION

FOR ADDING SOLUTIONS TOGETHER

[e] Solve the initial value problem $y'' - 2y' - 8y = 16x - 4, \quad y(0) = -1, \quad y'(0) = 12$.

$y(0) = \underline{1 + A + B = -1}$ ½ $\longrightarrow A + B = -2$

$y' = -2 + 4Ae^{4x} - 2Be^{-2x}$

$y'(0) = \underline{-2 + 4A - 2B = 12}$ ½ $\rightarrow 4A - 2B = 14$

$2A - B = 7$

$\longrightarrow 3A = 5 \longrightarrow \underline{A = \frac{5}{3}}$

$\underline{B = -\frac{11}{3}}$

$$\boxed{y = 1 - 2x + \frac{5}{3}e^{4x} - \frac{11}{3}e^{-2x}}$$

$y_1 = x$ is a solution of $x^2 y'' + x(x-2)y' + (2-x)y = 0$. Find a second linearly independent solution. **SCORE: _____ / 7 PTS**

$y_2 = vx$

$y_2' = v'x + v$

$y_2'' = v''x + 2v'$

$x^2 y_2'' + x(x-2)y_2' + (2-x)y_2$

$= \underline{x^2(v''x + 2v')} + \underline{x(x-2)(v'x+v)} + \underline{(2-x)vx}$

$= v''(x^3) + v'(2x^2 + x^2(x-2)) + v(x(x-2) + x(2-x))$

$= \underline{x^3 v'' + x^3 v' = 0}$

LET $u = v'$

$\cancel{x^3}u' + \cancel{x^3}u = 0$

$\frac{du}{dx} = -u$

$\int \frac{du}{u} = \int -dx$

$\ln|u| = -x$

$\underline{u = e^{-x}}$

$v' = e^{-x}$

$\underline{v = -e^{-x}}$

$\underline{y_2 = -xe^{-x} \text{ OR } xe^{-x}}$ EITHER ONE OK